AD-A158 762 SIEVES AND A FILTER FOR GAUSSIAN PROCESSES(U) WISCONSIN UNIV-MILWAUKEE J BEDER 25 APR 85 AFOSR-TR-85-0621 AFOSR-84-0329 1/1

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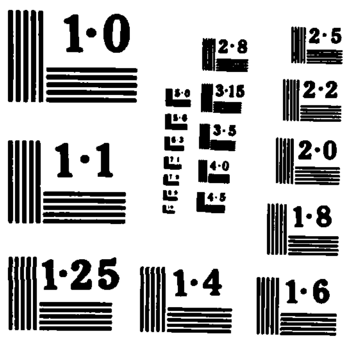

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Sieves and a Filter for Gaussian Processes: AFOSR-84-0329

Results on Conjecture 1.

We use the definitions and notation of [1]. Let H be the subspace of $L^2(\Omega,a,p)$ spanned by $\{X_t, t\varepsilon T\}$, and let $H = H(R,T)$ be the corresponding reproducing kernel Hilbert space. Assume that H has countable dimension, with CON basis $\{U_i\}$. Fix $\underline{a} = \{a_k\}\varepsilon \ell^2$, and consider a sample of size n.

Proposition 1. Under $P_{\underline{a}}$, the U_k are independent, and $\bar{U}_k$ has the $n(a_k, 1/n)$ distribution.

We consider the sieve estimator defined in [1], p. 7. Then for each m and for a sample of size n, the sieve estimator of the mean g(t) is

$$\hat{g}(t) = \Sigma \hat{a}_k g_k(t), \tag{1}$$

where $\{g_k\}$ is the CON basis of H corresponding to $\{U_k\}$. We wish to pick m as a function of n so that $\|\hat{g}-g\| \to 0$ (norm in H). But $\|\hat{g}-g\|^2 = \|\hat{\underline{a}}-\underline{a}\|^2$ (norm in ℓ^2) $= \sum_{k<m} (\bar{U}_k-a_k)^2 + \sum_{k>m} a_k^2 = X_{nm} + \sum_{k>m} a_k^2$, say. The non-stochastic tail vanishes as $n \to \infty$ if we require $m \to \infty$. By Proposition 1, nX_{nm} is $\chi^2(m)$, and the following weak result describes the limitations we will need to impose on $m = m_n$.

Proposition 2. If $m/n \to \beta$, then $X_{nm} \to 0$ in $P_{\underline{a}}$-probability iff $\beta=0$.

The Borel-Cantelli Lemma gives a sufficient condition for $P_{\underline{a}}(X_{nm}\to 0) = 1$:

Proposition 3. Let $\{m_n\}$ be chosen so that $m_n \to \infty$ and such that the sequence $\{c_n\}$ defined by

$$c_n = \int_{\frac{n\varepsilon}{2}}^{\infty} \frac{1}{\Gamma(\frac{m}{2})} u^{\frac{m}{2}-1} e^{-u}\, du$$

is summable for every $\varepsilon > 0$. Then $P_{\underline{a}}(X_{nm}\to 0) = 1$.

If m_n is chosen so that it is always even, then we may rewrite c_n as

$$c_n = e^{-\alpha n} e_{\ell-1}(\alpha n),$$

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Results are reported on the first two conjectures that were to be investigated as described in the research proposal. Conjecture 1 has been established and follow-on results are obtained. What remains to be investigated is the use of these results to make confidence statements and to test hypotheses. Results which establish the truth of conjecture 2 are also reported. Additional keywords: [illegible] estimates, covariance, convergence.

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where $\alpha = \varepsilon/2$, $n = 2\ell$, and $e_n(x) = \sum_{k=0}^{n} \frac{x^k}{k!}$. Invert $\ell = \ell(n)$, say $n = \lambda(\ell)$. Now we must choose $\lambda(\ell)$ so that at least $\lambda/\ell \to \infty$ and so that $e^{-\alpha\lambda} e_\ell(\alpha\lambda)$ is summable for every $\alpha > 0$. Write

$$e^{\alpha\lambda} = e_\ell(\alpha\lambda) + \frac{(\alpha\lambda)^\ell}{\ell!} A_\ell(\alpha).$$

Using results of Buckholtz [2], we may prove the following:

<u>Theorem 1</u>. Let $\lambda = \lambda(\ell)$ be such that $\lambda/\ell \to \infty$ as $\ell \to \infty$. Then for each $\alpha\varepsilon(0,\infty)$ we have

$$A_\ell(\alpha) = \frac{\ell! e^{\alpha\lambda}}{(\alpha\lambda)^\ell} + \frac{\alpha\lambda}{\ell - \alpha\lambda} + O(\frac{1}{\ell}), \quad \ell \to \infty .$$

From this and Stirling's formula we have

<u>Corollary 1.</u> If $\lambda/\ell \to \infty$ as $\ell \to \infty$, then for each $\alpha\varepsilon(0,\infty)$ the sequence

$$e^{-\alpha\lambda}\, e_\ell(\alpha\lambda)$$

is summable.

Translating back, we get the following consistency result.

<u>Theorem 2</u>. If $m = 2\ell(n) = o(n)$ and $m \to \infty$, then $\hat{\underline{a}} \to \underline{a}$ a.s.

At first this holds a.s. $P_{\underline{a}}$, but since the measures in P are equivalent, we may assert a.s. convergence without qualification. In any case, this settles Conjecture 1. In particular, the condition $m = o(n)$, which was sufficient for weak convergence, also gives strong convergence, and since $m/n \to \beta > 0$ does not even give weak convergence, our condition is in some sense best possible. Moreover, a result similar to Theorem 1 allows us to see just what does hold almost surely if we only require $m/n \to \beta > 0$:

Theorem 3. If $m/n \to \beta > 0$, then for all ε sufficiently large (depending only on β) we have

$$P_{\underline{a}}(X_{nm} > \varepsilon \text{ i.o.}) = 0$$

The distribution of the estimator $\hat{g}$ given by (1) is easy to describe. Consider $\hat{g}(t)$ as a stochastic process. From Proposition 1 we have the following.

Theorem 4. Under $P_{\underline{a}}$, the process $\{\hat{g}(t),\ t\varepsilon T\}$ is Gaussian with mean function $\sum_{i=1}^{m} a_i g_i(t)$ and covariance function $\frac{1}{n}\sum_{i=1}^{m} g_i(s)g_i(t)$.

For comparison we note that the reproducing kernel $R(s,t)$ of H may be written $R(s,t) = \sum_{i=1}^{\infty} g_i(s)g_i(t)$ (as long as $\{g_i\}$ is CON in H), and that under $P_{\underline{a}}$ the true mean function is $\sum_{i=1}^{\infty} a_i g_i(t)$. What needs to be investigated now is the use of Theorem 4 to make confidence statements and to test hypotheses.

Results on Conjecture 2.

The set-up here is given in [1], pp. 7-8. Let us fix a countably infinite orthonormal set $\{g_k\}$ (not necessarily complete) in H, and consider the subset $P_0 \subset P$ consisting of measures corresponding to covariances of from $S(s,t) = R(s,t) + \Sigma\mu_k g_k(s)g_k(t)$, where $\underline{\mu} = \{\mu_k\}$ is in the subset ℓ_e^2 of ℓ^2. Now the likelihood function depends only on $\underline{\mu}$. Let $U_k \varepsilon H$ correspond to g_k as usual, and let $A = \{\omega\varepsilon\Omega\colon U_k(\omega) \neq 0 \text{ for every } k\}$. Certainly A is an event in A.

Theorem 5. $Q(A) = 1$ for every $Q\varepsilon P$, and for each outcome $\omega\varepsilon\Omega$ the corresponding likelihood is unbounded over P_0. Thus Conjecture 2 is established.

References

[1] Jay H. Beder. Sieves and a filter for Gaussian processes, AFOSR grant proposal.

[2] J. D. Buckholtz. Concerning an approximation of Copson, *Proc. Amer. Math. Soc.* 14 (1961), 564-568.

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